



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

We further assume that $p > a < a \cot \frac{1}{2}C$. The solution would be just as simple with any other possible assumption.

On AB describe the segment ACB containing the given angle C . Draw the diameter PD perpendicular to AB . Also draw AQ perpendicular to AB . Draw BP meeting AQ in Q . With B as a center and a radius equal to p , describe an arc cutting AQ in R . With R as a center and a radius equal to $AQ - p$, describe an arc cutting AB produced in S . On SR measure off $SN = SA$. Then $RN =$ altitude required. Take $AL = RN$, and draw CL parallel to AB , cutting the circle in C . Draw AC , BC . Then ACB is the required triangle. Let x, y, z be the sides BC, AC , and the altitude. Then $xy \sin C = az$, $x + y - z = p$, $a^2 = x^2 + y^2 - 2xy \cos C$.

$$\therefore a^2 + 2xy(1 + \cos C) = (p + z)^2.$$

$$\therefore z^2 - 2(a \cot \frac{1}{2}C - p)z = a^2 - p^2.$$

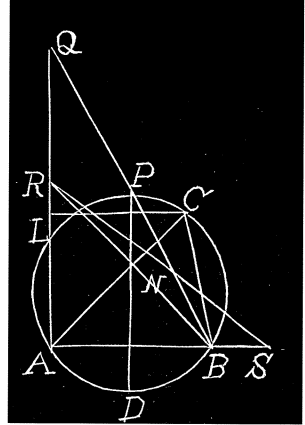
$$\therefore z = a \cot \frac{1}{2}C - p - \sqrt{[(a \cot \frac{1}{2}C - p)^2 - (p^2 - a^2)]}.$$

$$\angle AQB = \frac{1}{2}C, \quad AQ = a \cot \frac{1}{2}C, \quad RS = a \cot \frac{1}{2}C - p, \quad AR = \sqrt{[p^2 - a^2]},$$

$$AS = \sqrt{[(a \cot \frac{1}{2}C - p)^2 - (p^2 - a^2)]}.$$

$$\therefore RN = z. \quad \therefore \text{The triangle } ACB \text{ contains all the required parts.}$$

Also solved by J. Scheffer.



364. Proposed by R. C. ARCHIBALD, Providence, R. I.

Between the side of a given rhombus and its adjacent side produced, to insert a straight line of a given length and directed to the opposite corner. [“Euclidean constructions” are particularly desired.]

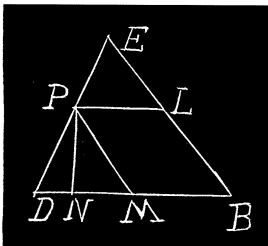
Solution by C. N. SCHMALL, New York City, and J. SCHEFFER, A. M., Hagerstown, Md.

This construction cannot be effected by Euclidean geometry. This will be evident from the algebraic analysis of the conditions.

Let $MBLP$ be the given rhombus, and P the given corner. Also let DE be the required line, so that $DE = l =$ given length. On AB drop the perpendicular PN . Now let $MB = BL = a$, $MN = c$, $DM = x$. Then we have, in $\triangle DMP$,

$$DP^2 = DM^2 + MP^2 - 2DM \cdot MN = x^2 + a^2 - 2cx.$$

$$\text{Also, } DM^2 : DP^2 = DB^2 : DE^2;$$



that is, $x^2 : (x^2 + a^2 - 2cx) = (x+a)^2 : d^2$.

$$\begin{aligned}\therefore d^2 &= \frac{(x+a)^2 (x^2 - 2cx + a^2)}{x^2} = (x+a)^2 \left(1 - \frac{2c}{x} + \frac{a^2}{x^2}\right) \\ &= x^2 + 2(a-c)x + 2a(a-2c) + 2a^2(a-c) \cdot \frac{1}{x} + \frac{a^4}{x^2}.\end{aligned}$$

Also solved by G. B. M. Zerr.

CALCULUS.

286. Proposed by R. D. CARMICHAEL, Princeton University.

Solve the differential equation

$$\begin{aligned}& [a_0x^3 + a_1x^2y + a_2xy^2 + (a_0 - a_1 + a_2)y^3 \\ & \quad + a_3x^2 + a_4xy + a_5y^2 + a_6x + a_7y + a_8]dx \\ & + [a_0y^3 + a_1xy^2 + a_2x^2y + (a_0 - a_1 + a_2)x^3 \\ & \quad + a_3y^2 + a_4xy + a_5x^2 + a_6y + a_7x + a_8]dy = 0.\end{aligned}$$

I. Solution by W. W. BEMAN, Professor of Mathematics, University of Michigan, Ann Arbor, Mich.

Putting $x=s+t$, $y=s-t$, and afterward $t^2=w$, the equation takes the linear form

$$\begin{aligned}\frac{dw}{ds} + \frac{4(3a_0 - 2a_1 + a_2)s + 2(a_3 - a_4 + a_5)}{4(a_1 - a_2)s^2 + 2(a_3 - a_5)s + a_6 - a_7} \cdot w \\ = - \frac{4(a_0 + a_2)s^3 + 2(a_3 + a_4 + a_5)s^2 + 2(a_6 + a_7) + 2a_8}{4(a_1 - a_2)s^2 + 2(a_3 - a_5)s + a_6 - a_7}.\end{aligned}$$

Or, putting $x+y=2u$, $xy=2v$, the equation takes the linear form

$$\begin{aligned}\frac{dv}{du} + \frac{4(3a_0 - 2a_1 + a_2)u + 2(a_3 - a_4 + a_5)}{4(a_1 - a_2)u^2 + 2(a_3 - a_5)u + a_6 - a_7} \cdot v \\ = \frac{8a_0u^3 + 4a_3u^2 + 2a_6u + a_8}{4(a_1 - a_2)u^2 + 2(a_3 - a_5)u + a_6 - a_7}.\end{aligned}$$

From each of these solutions it is obvious that the original equation has an integrating factor of the form $f(x+y)$.